OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4730**: Mechanics 3

Advanced GCE

Mark Scheme for June 2017

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Answer			Marks	Guidance	
1	**(i)**	Impulse/momentum triangle with sides 0.8, 1.2 and 1	B1	OR $1.2\cos\alpha = \cos\theta - 0.8$ $1.2\sin\alpha = \sin\theta$ M1	
		$\cos\theta = \dfrac{0.8^2 + 1^2 - 1.2^2}{2 \times 0.8 \times 1}$	M1	$1.44 = (\cos\theta - 0.8)^2 + \sin^2\theta$ A1	Square and add
		82.8° or 1.44 rads	A1	isw	82.81924° or 1.445 rads
		$1.2\sin\alpha = \sin\theta$	M1	cv θ; OR from cos rule	may see 55.771° or 0.97339 rads
		Angle 124°	A1 [5]	No isw	2.168 rads
2	**(i)**	$\frac{1}{2}m \times 0.7^2 = \frac{1}{2}mv^2 + \frac{24mg0.3^2}{2 \times 1.2} - mg \times 1.5$	M1	By energy; needs KE, PE and EE terms	Allow wrong signs, missing '2'
			A1	OR $\frac{1}{2}m \times 4.9^2 = \frac{1}{2}mv^2 + \frac{24mg0.3^2}{2\times1.2} - mg \times 0.3$	
		Speed = 3.5 (ms^{-1})	A1 [3]	**AG** Adequate working, no errors	
	(ii)	One correct EE term involving x seen	B1		
		$\frac{1}{2}m \times 0.7^2 = \dfrac{24mg(x-1.2)^2}{2 \times 1.2} + \dfrac{32mg(x-1.5)^2}{2 \times 0.8} - mgx$	M1	Where x is distance below O OR, where x is dist from T, $\frac{1}{2}m \times 0.7^2 = \frac{24mg(x+0.3)^2}{2\times1.2} + \frac{32mgx^2}{2\times0.8} - mg(x+1.5)$	Energy equation with at least 1 KE, 1 PE and 1 EE term and values subst. Alt left side: $\frac{1}{2}m3.5^2 + \frac{24mg0.3^2}{2\times1.2} - 1.5mg$
			A1	Leads to $48x^2 + 8x - 1 = 0$	
		$[48x^2 - 136x + 95 = 0]$	M1	Correct attempt to solve their 3 term quad.	Dep M1 above
		1.25 (m) and 1.58 (m)	A1 [5]	$1\frac{1}{4}$ $1\frac{7}{12}$	

3	**(i)**	48 (ms^{-1})	B1 [1]	Accept ≤ 48	
	(ii)	Use $\frac{1}{2}\sqrt{12-\frac{1}{4}v}=0.2a$	M1*	Accept $v\frac{dv}{dx}$ for a	Allow missing 0.2 or sign error
		$\frac{1}{2}\sqrt{12-\frac{1}{4}v}=0.2\frac{dv}{dt}$	A1		
		$2.5t=\int\frac{dv}{\sqrt{12-\frac{1}{4}v}}(+c)$	*M1*	Sep variables and integrate one side	$2.5t=\int\frac{2dv}{\sqrt{48-v}}(+c)$
		$2.5t=-8\left(12-\frac{1}{4}v\right)^{\frac{1}{2}}(+c)$	A1		$2.5t=-4(48-v)^{\frac{1}{2}}(+c)$
		[c = 24]	*M1	For attempt to find c, dep previous M1	
		$v=48-4\left(3-\frac{t}{3.2}\right)^2$	A1 [6]	oe $12+7.5t-\frac{25}{64}t^2$ (0.390625)	$v=48-0.390625(9.6-t)^2$
	(iii)	$x=\int(12+\frac{24}{3.2}t-\frac{4}{3.2^2}t^2)dt$	M1	OR $x=\int(48-4\left(3-\frac{t}{3.2}\right)^2)dt$	OR $\frac{1}{2}\sqrt{12-\frac{1}{4}v}=0.2v\frac{dv}{dx}$
		$x=12t+3.75t^2-0.1302t^3(+c)$	A1	$x=48t+\frac{12.8}{3}\left(3-\frac{t}{3.2}\right)^3(+c')$	via subst $\left(12-\frac{1}{4}v\right)=u^2$
		(t = 0 and) t = 3.2	M1	ft their (ii)	$x=12.8\left(12u-\frac{u^3}{3}\right)+C$
		Distance = 72.533 (m)	A1 [4]		

4	**(i)**	Momentum equation $2ma = -2m\frac{1}{10}\sqrt{5}\frac{1}{\sqrt{5}} + 7m\frac{1}{10}\sqrt{5}\frac{1}{\sqrt{5}}$ $(a =)\frac{1}{4}$ (ms^{-1}) Comp of speed of A perp = 0.2 Speed of A was $\sqrt{(0.25^2 + 0.2^2)}$ OR $\tan\theta = \frac{0.2}{0.25}$ Speed 0.320 or $\frac{\sqrt{41}}{20}$; Ang 38.7° or 0.675 rads NLM $0.1 + 0.1 = -e(0 - a))$ (e =) 0.8	M1 A1 B1 M1 A1 M1 A1 [7]	Along line of centres Allow errors with signs and masses soi soi Allow their vel comps oe For both angle and speed Along line of centres Allow errors with signs	Allow use of 63.4° for full marks Must use comp of vel 0.320156; 38.6598° or 0.67474 rads May see $\frac{1}{10}\sqrt{5}\frac{1}{\sqrt{5}}$ for 0.1
	(ii)	A and B have same speed perpendicular to line of centres after first collision	B1 [1]	accept 'vertical'	
	(iii)	Momentum equation along line of centres $3mU - 2m\frac{1}{10}\sqrt{5}\frac{1}{\sqrt{5}} = 3mc + 2ma'$ NLM $a' - c = -1(-\frac{1}{10}\sqrt{5}\frac{1}{\sqrt{5}} - U)$ Use $a' = 0.1$ Max $U = \frac{1}{15}$	M1 A1 M1 A1 B1 A1 [6]	Allow errors with signs and masses Allow cosα for $\frac{1}{\sqrt{5}}$ Allow errors with signs Accept any inequality Accept 0.0667 accept $\leq$	Must use comp of vel Or conservation of energy $\frac{1}{2}3mU^2 + \frac{1}{2}2m0.1^2 = \frac{1}{2}3mc^2 + \frac{1}{2}2ma'^2$ do not accept $<$
5	**(i)**	$3mga\cos\frac{\pi}{6}$ and $2mga\cos\frac{\pi}{6}$ $3mga\cos\left(\frac{\pi}{6}+\theta\right) + 2mga\cos\left(\frac{\pi}{6}-\theta\right) + \frac{1}{2}3mv^2 + \frac{1}{2}2mv^2$ $v^2 = \frac{2}{5}ag\left(5\cos\frac{\pi}{6} - 3\cos\left(\frac{\pi}{6}+\theta\right) - 2\cos\left(\frac{\pi}{6}-\theta\right)\right)$	B1 M1 A1 A1 [4]	Initial PE Final PE + KE **AG** Equating and correct manipulation	If O is zero level for PE For M1 at least 1 KE and 1 PE term; allow m used for 2m/3m; wrong signs; missing g

	(ii)	$v^2 = \frac{2}{5}ag\left(5\cos\frac{\pi}{6} - 3\cos\frac{\pi}{3} - 2\cos 0\right)$ $3mg\cos\frac{\pi}{3} - R = 3m\frac{v^2}{a}$ $R = 3mg\cos\frac{\pi}{3} - 3m\frac{2g}{5}\left(5\cos\frac{\pi}{6} - \frac{7}{2}\right)$ $R = mg\left(5.7 - 6\cos\frac{\pi}{6}\right)$ oe	B1 M1 A1 A1 [4]	$v^2 = \frac{1}{5}ag(5\sqrt{3} - 7)$ *F = ma*, condone sign error; allow m used for 2m/3m Accept 0.5038475*mg* or $mg(5.7 - 3\sqrt{3})$ oe	OR $3mg\cos\left(\frac{\pi}{6} + \theta\right) - R = 3m\frac{v^2}{a}$ $R = 3mg\cos\left(\frac{\pi}{6} + \theta\right) -$ $3m\frac{2}{5}g\left(5\cos\frac{\pi}{6} - 3\cos\left(\frac{\pi}{6} + \theta\right) -$ $2\cos\left(\frac{\pi}{6} - \theta\right)\right)$ $R = mg(6.6\cos\left(\frac{\pi}{6} + \theta\right) +$ $2.4\cos\left(\frac{\pi}{6} - \theta\right) - 6\cos\frac{\pi}{6})$ Answer must be simplified 4.94*m* loses last mark
6	**(i)**	$Pl\sqrt{5} = W \times 3l\cos\theta$ $P = 1.2W$ $Ql\sqrt{2} = U \times \frac{\lambda}{2}l\cos\emptyset$ $Q = 0.25\lambda U$	M1 A1 M1 A1 [4]	Mom about *A* for *AB* **AG** Mom about A for AC	Allow sin θ, cancelled *l* Not from use of angle 26.565° Allow sin $\emptyset$, cancelled *l*
	(ii)	(H) $P\sin\theta = Q\sin\emptyset$ (V) $W + U = P\cos\theta + Q\cos\emptyset$ $W + U = P\cos\theta + P\sin\theta \times \frac{\cos\emptyset}{\sin\emptyset}$ $W + U = \frac{3}{\sqrt{5}} \times 1.2W$ $k = 0.610$ $\lambda = 4.98$ $[P\sqrt{5}l - W3l\cos\theta = Q\sqrt{2}l - U\frac{\lambda}{2}\cos\emptyset$	M1 M1 A1 M1* *M1 A1 A1 [7]	$P\frac{1}{\sqrt{5}} = Q\frac{1}{\sqrt{2}}$; compts essential $W + U = P\frac{2}{\sqrt{5}} + Q\frac{1}{\sqrt{2}}$; compts essential Eliminate *Q* (or *P*) dep M1M1 Elim P and Q to get equation in *k*, $W + U = 1.609689W$ $\frac{18\sqrt{5} - 25}{25}$ Mom about A (or any other point) for whole system – allow M1(A1) if resolving not seen twice]	Allow $\frac{2}{\sqrt{5}}$ for M1 Allow $\frac{1}{\sqrt{5}}$ for M1, sign errors $W + U = Q\cos\theta \times \frac{\sin\emptyset}{\sin\theta} + Q\cos\emptyset$ $\left[W + U = 0.25\lambda U \times \frac{3}{2\sqrt{2}}\right]$ 0.6099689 4.97695 Allow use of angles in (ii): 26.6 & 45 OR after M1M1A0/1, M1* for 2 equns in terms of k and λ, *M1 for solving for k or λ.

7	(i)	$\frac{1}{2}m \times \frac{g}{90} = mgh$ [Max height = $\frac{1}{180}$ = 0.005556] Max angle = 6.76° or 0.118 rads $-mg\sin\theta = m \times 0.8 \times \ddot{\theta}$ $\ddot{\theta} = -\frac{9.8}{0.8}\theta$, SHM (about θ = 0) since θ is small $\omega^2 = 12.25$ Period = 1.80 secs $(\frac{4}{7}\pi)$	M1 A1 M1 A1 A1 M1 A1 [7]	By energy; allow cancelled *m* Allow M1A1 for 6.76° or 0.118 rads in (ii) N2L; allow *a* for 0.8 $\ddot{\theta}$; allow cancelled *m* Cand value	$\frac{1}{2}m \times \frac{g}{90} = mg \times 0.8(1-\cos\theta)$ 6.756 / 0.11798 allow sign error, sin / cos 1.7952
	(ii)	$0.087266 = A\sin 3.5t$ *t* = 0.238 secs $t' = 2\left(\frac{1.7952}{4} - 0.2378\right) = 0.422$(s) $\dot{\theta} = 0.118 \times 3.5\cos 3.5 \times 0.238$ Linear speed = 0.222(ms^{-1})	M1 A1 A1 M1 A1 [5]	OR $5 = A\sin 3.5t$; *A* = amplitude Or 0.65972 Or 0.65972 – 0.2378 OR $\dot{\theta} = \sqrt{(3.5^2(0.118^2 - 0.0873^2)}$ 0.8 ×0.278	May use cos * 0.2378 allow sin if consistent with *; allow 5° and 6.76° Or $\frac{1}{2}m\frac{9}{90} = \frac{1}{2}mv^2 + mg0.8(1-\cos 5°)$
	(iii)	Max height is still 0.00556 soi Max angle = $\cos^{-1}\frac{(0.05-0.00556)}{0.05}$ [27.3] Not SHM since angle is not small	B1 B1 B1 [3]	accept 'still the same'	or attempt to work out height 0.476 rads

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